AF-3569

M.A./M.Sc. (Final)
Term End Examination, 2017-18

MATHEMATICS

Paper - IX

Fuzzy Sets and Their Applications

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) Explain why we need Fuzzy set theory.

(*b*) Prove that a Fuzzy set A on R is convex if and only if $A(\lambda x_1 + (1 - \lambda) x_2) \geq \min [A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0, 1]$, where min denotes the minimum operator.

2. (*a*) Compute the scalar cardinalities for each of the following fuzzy sets :

(*i*) $A = \frac{.4}{v} + \frac{.2}{w} + \frac{.5}{x} + \frac{.4}{y} + \frac{1}{z}$

(*ii*) $D(x) = 1 - \frac{x}{10}$ for $x \in \{0, 1, \ldots.., 10\} = X$

(*iii*) $B = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

(*iv*) $C(x) = \frac{x}{x+1}$ for $x \in \{0, 1, 2, ..., 10\} = X$

(*b*) Let A, B $\in F$ (α). Then prove that $\forall\, \alpha \in [0,\ 1]$

(*i*) ${}^{\alpha+}A \subseteq {}^{\alpha}A$

(*ii*) ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

3. (*a*) Consider two triangular shape Fuzzy numbers A and B as follows :

$$A(x) = \begin{cases} 0, & \text{for } x \le -1 \text{ and } x > 3 \\ \dfrac{x+1}{2}, & \text{for } -1 < x \le 1 \\ \dfrac{3-x}{2}, & \text{for } 1 \le x \le 3 \end{cases}$$

Find the Fuzzy number $A + B$.

(*b*) Define t-norms. Prove that the standard Fuzzy intersection is the only idempotent t-norm.

4. (*a*) Define fuzzy numbers ansd show that

$$\sup_{z=x*y} \min\left[A(x), B(y)\right]$$

is a continuous Fuzzy number.

(*b*) Let i be a t-norm such that $i(a, b+c) = i(a, b) + i(a, c)$ for all a, b, $c \in [0, 1]$, $b + c \leq 1$. Show that i must be the algebraic product; that is $i(a, b) = a \,.\, b$ for all a, $b \in [0, 1]$.

5. (*a*) For the following binary relation

$$R = \begin{bmatrix} 1 & 0 & .7 \\ 0 & 1 & 0 \\ .7 & 0 & 1 \end{bmatrix}$$

Find the domain, range, height and inverse of the relation R.

(*b*) For any Fuzzy relation R on x^2, the Fuzzy relation

$$R_{T(i)} = \bigcup_{n=1}^{\infty} R^{(n)}$$

is the i-transtitive closure of R. Show it.

6. (*a*) Solve the following Fuzzy relation using max-min composition :

$$p_{\circ} = \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = [.6 .6 .5]$$

(*b*) Define Fuzzy equivalence relation and illustrate with one example.

7. (*a*) Define body of evidence, total ignorance, focal element and joint probability.

(*b*) Let $x = \{a, b, c, d\}$. Given the basic assignment $m(\{a, b, c\}) = .5$, $m(\{a, b, d\}) = .2$ and $m(x) = 0.3$ determine the corresponding belief and plausibility measures.

8. (*a*) Explain linguistic hedges and consider some linguistic hedges and determine reasonable modifiers for them.

(*b*) Explain unconditional and qualified propositions with suitable example.

9. (*a*) Explain the architecture of an expert system.

(*b*) Define Fuzzy implication with at least one example and also write the reasonable axioms of fuzzy implications.

10. (*a*) What is the role of defuzzification in fuzzy controllers ? Discuss any two defuzzification methods.

(*b*) Assume that each individual of a group of eight decision makers has a total preference ordering p_i $(i \in N_8)$ on a set of alternatives $X = (w, x, y, z)$ as follows :

$P_1 = (w, x, y, z)$, $P_2 = P_5 = (z, y, x, w)$

$P_3 = P_7 = (x, w, y, z)$, $P_4 = P_8 = (w, z, x, y)$

$P_6 = (z, w, x, y)$

use multiperson decision making to determine the group decision.