# AF-3558

M.A./M.Sc. (Previous)
Term End Examination, 2017-18

## MATHEMATICS

Paper - IV

Complex Analysis

*Time* : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

**Note** : Answer any **five** questions. Answer to each question should begain on a fresh page. All questions carry equal marks.

**1.** (*a*) State and prove Cauchy's integral formula.

(*b*) State and prove Liouville's theorem.

**2.** (*a*) Prove that the function $\sin c\left(z+\frac{1}{z}\right)$ can be expanded in a series of the type $\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^n$ in which the coefficients of both $z^n$ and $z^{-n}$ are

$$\frac{1}{2\pi}\int_0^{2\pi} \sin\left(2c\cos\theta\right)\cos n\theta \, d\theta .$$

(*b*) State and prove Argument Principle.

**3.** (*a*) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

(*b*) State and prove Maximum modulus Principle.

**4.** (*a*) (*i*) Find the singularities of

$$f\left(z\right) = \frac{\left(z-2\right)}{z^2}\sin\left(\frac{1}{z-1}\right)$$

(*ii*) Find the residue of $\frac{z^3}{z^2-1}$ at $z = \infty$.

(*b*) Prove that :

$$\int_0^\infty \frac{\cos mx}{a^2 + x^2}\,dx = \frac{\pi}{2a} e^{-ma} \left(m \geq 0\right)$$

using method of contour integration.

**5.** (*a*) (*i*) Write all the critical points and fixed points of bilinear transformation

$$w = \frac{az + b}{cz + d}\left(ad - bc \neq 0\right).$$

(*ii*) Find the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$ and $z_3 = -2$ into the points $w_1 = 2$, $w_2 = \mathrm{i}$ and $w_3 = -1$.

(*b*) Let $f(z)$ be an analytic function of $z$ in a region $D$ of the z-plane and $f'(z) \neq 0$ inside $D$. Then prove that the mapping $w = f(z)$ is conformal at the points of $D$.

**6.** (*a*) State and prove Hurwitz theorem.

(*b*) Explain any **two** of the following :

(*i*) Normal Set and Normal Family

(*ii*) Totally Bounded Set

(*iii*) Equicontinuity

**7.** (*a*) Find residue of $\Gamma z$ (gamma $z$) at the poles.

(*b*) Prove that $\log \sqrt{z}$ is a convex function on $(0, \infty)$.

**8.** (*a*) State and prove Runge's theorem.

(*b*) Prove that $\operatorname{cosec} z = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2\pi^2 - z^2}$.

**9.** (*a*) State and prove Schwartz's Reflection principle.

(*b*) Let $f$ be an analytic function on a region containing $\overline{B}(0, r)$ and suppose that $a_1$, $a_2$, .... $a_n$ are the zeros of $f$ in $\overline{B}(0, r)$ repeated according to multiplicity. If $f(0) \neq 0$, then prove that

$$\log|f(0)| = -\sum_{k=1}^{n} \log\left(\frac{r}{|a_k|}\right) + \frac{1}{2\pi}\int_0^{2\pi} \log\left|f\left(re^{i\theta}\right)\right| d\theta$$

**10.** (*a*) State and prove Harnack's inequality.

(*b*) State and prove Schottky's theorem.