AF-3560

M.A./M.Sc. (Previous)
Term End Examination, 2017-18

MATHEMATICS

Paper - VI

Differential Geometry of Manifold's

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. Let X be vector field on the smooth manifold M. Then the lie derivative L_x is the unique derivation of $\int_*^*(M)$ with the following properties :

(*a*) $L_x f = \langle df, x \rangle = Xf, \forall f \in c^{\infty}(M)$

(*b*) $L_x Y = [X, Y] \forall X, Y \in \text{Vect}(M)$

2. (*a*) Let M be a differentiable n-manifold and let P be any point in M. Prove that $T_p(M)$ is an n-dimensional vector space.

(*b*) Set and prove Local Immersion theorem.

3. (*a*) Show that the tangent bundle is a vector bundle.

(*b*) Show that the range of the zero section of a vector bundle $E \to M$ is a submanifold of E.

4. (*a*) Show that for each positive integer K the space R^K is a differential manifold.

(*b*) Prove that $X^T = K_M \circ TX$ for vector bundle homomorphism.

5. (*a*) State and prove Schur's Theorem.

(*b*) Define the following with example.

(*i*) Nijenhuis tensor

(*ii*) Conformal curvature tensor

(*iii*) Exterior derivative

(*iv*) Bundle homomorphism

6. (*a*) Let G be a Lie group and H a subgroup which is also a regular submanifold. Then with its differentiable structure as a submanifold H is a Lie group.

(*b*) If H is a regular submanifold and subgroup of a Lie group G, then prove that it is closed as a subset of G.

7. If $F : G_1 \to G_2$ is a homomorphism of Lie groups, then prove that :

(*a*) Rank of F is constant

(*b*) Kernel of F is Lie group

(*c*) $\dim(\text{Ker } F) = \dim G_1 - \text{rank } F$

8. State and prove Generalized Gauss and Mainardi-Codazzi equations.

9. (*a*) Prove that every vector bundle of dimension n over V is associated to a principal bundle over V with group $GL(n, R)$.

(*b*) Define the following :

(*i*) Tangent bundle

(*ii*) Induced bundle

(*iii*) Principle fibre bundle

10. (*a*) Prove that Riemannian geodesic is Locally minimizing.

(*b*) State and prove First variation formulae.