AF-3559

M.A./M.Sc. (Previous)
Term End Examination, 2017-18

MATHEMATICS

Paper - V

Advanced Discrete Mathematics

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.

1. (*a*) Prove that the following is tautology :

$$(p \to q) \to \left[p \vee (q \wedge r) \leftrightarrow q \wedge (p \vee r)\right]$$

(*b*) Show that the following argument is valid

$$\begin{array}{c} p \\ p \to q \\ \underline{q \to r} \\ r \end{array}$$

2. (*a*) Prove that direct product of two semigroups is a semigroup.

(*b*) If $(M, *)$ is a commutative monoid then the set of all idempotent elements of M forms a submonoid.

3. (*a*) State and prove the fundamental theorem of homomorphism for semigroups.

(*b*) Let $(M, *)$ be a monoid with identity element e and $(T, 0)$ be any algebraic structure. If $f : M \to T$ is onto and satisfies

$$f(a * b) = f(a) \bullet f(b) \ \forall \ a, b \in M$$

Then prove that $(T, 0)$ is monoid with $f(e)$ as its identity element.

4. (*a*) Prove that in distributive lattice the complement of an element is unique if it exist.

(*b*) Let $(L \leq)$ is a lattice. Then prove that

$$a \vee (b \vee c) = (a \vee b) \vee c \ \forall a, b, c \in L$$

where $a \vee b = \sup\{a, b\}$

5. (*a*) In Boolean Algebra B prove that

$$(a+b)' = a' \cdot b' \ \forall a, b \in B$$

(*b*) In Boolean Algebra B prove that

$$pqr' + pq'r + p'qr = pq + qr + pr$$

$$\forall p, q, r \in B$$

6. (*a*) Express the following function into disjunctive normal form

$$f(x, y, z) = (x+y+z)(xy + x'z)'$$

(*b*) Draw the logic circuit for each of the following expressions :

(*i*) $f = (x+y)(x'+y'+z')(y' \cdot z')$

(*ii*) $f = xyz + x'z' + y'z'$

7. (*a*) Construct a grammar generating the language

$$1 = \left\{a^n b^n c^i : n \geq 1 \ i \geq 0\right\}$$

(*b*) Describe the type of grammar in detail.

8. (*a*) Design a finite state machine which can add two binary numbers.

(*b*) State and prove pumping lemma.

9. (*a*) Prove that number of odd vertices in a graph is always even.

(*b*) Prove that a simple graph with n vertices and k components can have at most

$$\frac{(n-k)(n-k+1)}{2} \text{ edges}.$$

10. (*a*) Prove that a connected graph is an Euler graph if and only if the degree of every vertex in graph is even.

(*b*) Show that the maximum number of edges in a complete bipartite graph of n vertices is $\frac{n^2}{4}$.

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