AF-3556

M.A./M.Sc. (Previous)
Term End Examination, 2017-18

MATHEMATICS

Paper - II

Real Analysis and Measure Theory

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) Let $f, \ \alpha : [a, b] \to \mathbb{R}$ be a bounded function and α be monotonically increasing. If P^* is a refinement of the partition P of the interval $[a, b]$, then

$$L(P, f, \alpha) \le L(P^*, f, \alpha) \text{ and}$$
$$U(P^*, f, \alpha) \le (U, P, f, \alpha)$$

(*b*) Let F and G be differentiable function on $[a, b]$, $F' = f \in \mathbb{R}$ $[a, b]$ and $G' = g \in \mathbb{R}$ $[a, b]$. Then

$$\int_a^b F(x)g(x)\,dx = F(b)G(b) - F(a)G(a)$$

$$-\int_a^b f(x)G(x)\,dx.$$

2. (*a*) State and prove Riemann's theorem on rearrangement of series.

(*b*) Prove that the series :

$$1+\frac{1}{2}+\frac{1}{3}-1+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}-\frac{1}{2}+\ldots\ldots = \log 3$$

3. (*a*) Let (X, d) be metric space f be a function from X to $\mathbb{R}$ and $fn : X \to \mathbb{R}$ all $n \in \mathbb{N}$ the sequence of function $\{fn\}$ converges pointwise to f if and only if for each $x \in X$ and for each $\in > 0$ $\exists$ a +ve integer m such that $n \geq m \Rightarrow$

$$|fn(x) - f(x)| < \in.$$

(*b*) State and prove Mn Test for uniform convergence of sequence.

4. (*a*) Test for uniform convergence and continuity of the sum function of the series of which

$$fn\,(x) = \frac{1}{1+nx} \quad \text{for } 0 \le x \le 1$$

(*b*) Let α be monotonically increasing on $[a, b]$. Suppose $fn \in \mathbb{R}(\alpha)$ on $[a, b]$ for $n = 1, 2, 3, ...$ and let $fn \to f$ uniformly on $[a, b]$. Then $f \in \mathbb{R}(\alpha)$ on $[a, b]$ and

$$\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b fn\, d\alpha.$$

5. (*a*) State and prove Weierstrass's approximation theorem.

(*b*) Show that the power series $1 + 2x + 3x^2 + 4x^3 + ...$ has radius of convergence equal to one.

6. (*a*) State and prove Abel's theorem on power series.

(*b*) A linear operator A on a finite dimensional vector space X is one to one if and only if the range of A is all of X that is iff A is onto.

7. (*a*) State and prove Taylor's theorem.

(*b*) Prove that a Borel measurable set is Lebesgue measurable.

8. If E_1 and E_2 are non-measurable sets, then prove that

$$m(E_1 - E_2) = m(E_1) - m(E_2)$$

where $E_1 \supset E_2$ and $m(E_2) < \infty$.

9. (*a*) State and prove Lebesgue bounded convergence theorem.

(*b*) Evaluate the Lebesgue integral of function $f : [0, 1] \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \dfrac{1}{x^{3/2}} & \text{if} \quad 0 < x \leq 1 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

and show that f is Lebsesgue integrable on $[0, 1]$.

10. (*a*) Let $f, g \in L^P [a, b]$. Then $f + g \in L^P [a, b]$.

(*b*) State and prove Minknowski's inequality.

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