AF-3561

M.A./M.Sc. (Final)
Term End Examination, 2017-18

MATHEMATICS

Paper - I

Integration Theory and Functional Analysis

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) Prove that a union of countable collection of positive subset of X is positive.

(*b*) Let E and F are measurable sets and μ is a signed measure such that $E \subset F$ and $|\mu(F)| < \infty$. Then prove that $|\mu(E)| < \infty$.

2. (*a*) Using Fubini's theorem prove that

$$\int_0^1\left[\int_0^1 \frac{x^2-y^2}{\left(x^2+y^2\right)^2}\,dx\right]dy \neq \int_0^1\left[\int_0^1 \frac{x^2-y^2}{\left(x^2+y^2\right)}\,dy\right]dx$$

(*b*) If $(X, A_1\ \mu)$ and (Y, A_2, v) are measure spaces, then there exist measure π defined on $A = A_1 \times A_2$ such that $\pi(A \times B) = \mu(A)\ v(B)$ for all $A \in A$ and $B \in A_2$.

3. (*a*) If a function f is absolutely continuous in an interval (a, b) and $f'(x) = 0$ *a.e.* in $[a, b]$, then f is constant.

(*b*) If X and Y are locally compact T_2 space and A_0, B_0, S_0 are σ-ring of Baire sets in X, Y, $X \times Y$ respectively, then prove that $S_0 = A_0 \times B_0$

4. (*a*) Prove that in a normed linear space every convergent sequence is a Cauchy sequence.

(*b*) Prove that a non-zero normed linear space N is a Banach space if and only if $S = \{x : \|x\| = 1\}$ is complete.

5. (*a*) Let N and N' be normed linear spaces over the same scalar field and T be a linear transformation of N into N'. Then prove that T is bounded if and only if it is continuous.

(*b*) Prove that every complete subspace of a normed linear space is closed.

6. (*a*) Show that a normed linear space is separable if its conjugate space is separable.

(*b*) A non-empty subset X of a normed linear space N is bounded if and only if $f(x)$ is bounded set of numbers for each $f \in N^*$.

7. (*a*) State and prove Bessel's inequality in Hilbert space.

(*b*) Let S be a non-empty subset of a Hilbert space H. Then prove that $S^{\perp}$ is a closed linear subspace of H.

8. State and prove Riesz Representation theorem.

9. Prove that adjoint operator $T \to T^*$ has the following properties :

(*i*) $(T_1 + T_2)^* = T_1^* + T_2^*$

(*ii*) $(T_1 T_2)^* = T_2^* T_1^*$

(*iii*) $\|T^*\| = \|T\|$

(*iv*) $\|T^* T\| = \|T\|^2$

10. (*a*) If N is a normal operator on a Hilbert space H, then prove that $\|N^2\| = \|N\|^2$.

(*b*) An operator T on a Hilbert space H is self-adjoint if and only if (T_x, x) is real for all $x \in H$.