AF-3565

M.A./M.Sc. (Final)
Term End Examination, 2017-18

MATHEMATICS

Paper - V

Difference Equations

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) If $y(x)$ is a polynomial of n^{th} degree, then its n^{th} differences are constant and the $(n+1)^{\text{th}}$ differences are zero.

(*b*) Show that :

$$ {}^{n+1}C_1 u_0 + {}^{n+1}C_2 \Delta u_0 + {}^{n+1}C_3 \Delta^2 u_0 + \cdots + \Delta^n u_0 = u_0 + u_1 + u_2 + \cdots u_n $$

2. (*a*) If $\langle y_x \rangle$ is the solution sequence of the difference equation $y_{x+1} = y_x + B$, $x = 0$, 1, 2 with y_0 prescribed, then $\langle y_x \rangle$ is a constant sequence, if $B = 0$, it diverges to $+\infty$, if $B > 0$ and diverges to $-\infty$, if $B < 0$.

(*b*) Solve that :

$(E - a)(E - b)\, y_x = 0$, if $a \neq b$

3. (*a*) Solve that :

$$y_{x+1} - y_x = 1$$

(*b*) Solve that :

$$\Delta y_x + \Delta^2 y_x = \sin x$$

4. (*a*) If $Z\,[\langle f(x) \rangle] = F(Z)$, then

$$Z\left[\left\langle nf(x)\right\rangle\right] = {-ZdF(z)}\big/{dz}$$

(*b*) Using the inversion integral method, find the inverse Z-transfer of $\dfrac{3z}{(z-1)(z-2)}$

5. (*a*) Evaluate the sum $1^2 + 2^2 + 3^2 + \cdots + r^2$.

(*b*) Solve the difference equation $a_r = 8a_{r-1} + 10^{r-1}$, with the initial condition $a_0 = 1$.

6. (*a*) The condition $1 + C_1 + C_2 > 0$, $1 - C_1 + C_2 > 0$, $1 - C_2 > 0$, are necessary and sufficient for both roots of $m_2 + C_1 m + C_2 = 0$ to be less than 1 in absolute value.

(*b*) Find the critical point of the system

$\frac{dx}{dt} = -y$, $\frac{dy}{dt} = x$ and discuss its nature

and stability. Also find the general solution of the system and sketch the trajectories.

7. (*a*) State and prove the Lagrange's identity.

(*b*) Let f and g be real linearly independent solution of

$$\frac{d}{dx}\left[P(x)\frac{dy}{dx}\right] + Q(x)y = 0$$

on $x \in [a, b]$. Then between any two consecutive zeros of f, there is precisely one zero of g.

8. (*a*) Find non-trivial solution of the

Sturm-Liouville problem $\frac{d^2y}{dx^2} + xy = 0$.

(*b*) Find the formal expansion of the function f equation, where $f(x) = \pi x - x^2$, $0 \le x \le \pi$, in the series of orthonormal characteristic function $\{g_n\}$ of the Sturm-Liouville problem

$$\frac{d^2 y}{dx^2} + \lambda y = 0, \text{ with } y(0) = 0, \ y(\pi) = 0.$$

9. (*a*) Prove that a necessary condition for $\int_{x_1}^{x_2} f(x, y, y')\, dx$, to be an extremum is that

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{df}{dy'}\right) = 0$$

(*b*) Show that the shortest curve joining two fixed points is a straight line.

10. (*a*) Transform the equation $y'' + \frac{1}{Z} y' + \left(\lambda - \frac{N^2}{Z^2}\right)$, into normal form.

(*b*) Find the plane curve of fixed perimeter and maximum area.

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