AF-3557

M.A./M.Sc. (Previous)
Term End Examination, 2017-18

MATHEMATICS

Paper - III

Topology

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) If X is a non-empty countable set and T is a collection consisting of the empty set and all those subsets of X whose complements are countable, then T is a topology on X.

(*b*) Let T_1 and T_2 be two topologies on a non-empty set X, then show that $T_1 \cap T_2$ is also a topology on X, but $T_1 \cup T_2$ is not necessarily a topology on X.

2. (*a*) Given a non-empty set X and A, $B \subset X$ being arbitrary, if the Kuratowski closure operators have the properties

(*i*) $\overline{\phi} = \phi$,

(*ii*) $A \subseteq \overline{A}$,

(*iii*) $\overline{\overline{A}} = \overline{A}$,

(*iv*) $\overline{A \cup B} = \overline{A} \cup \overline{B}$,

then there exists a unique topology T on X such that $\forall A \subset X$, A coincides with T-closure of A.

(*b*) Show that a homeomorphic image of a first countable space is first countable.

3. (*a*) Prove that every second countable space is a Lindelof space.

(*b*) Let (X, T) be a topological space and $Y \subset X$, then show that the collection $T_0 = \{A \cap Y : A \in T\}$ is a topology on Y.

4. (*a*) Prove that every metric space is a Hausdorff Space.

(*b*) Let F_1, F_2 be any pair of disjoint closed sets in a normal space X. Then $\exists$ a continuous map $f : X \to [0, 1]$, such that $f(x) = 0$, for $x \in F_1$ and $f(x) = 1$ for $x \in F_2$.

5. (*a*) Prove that any closed subset (subspace) of a compact space is compact.

(*b*) Show that a sequentially compact topological space (X, T) is countably compact.

6. (*a*) Prove that a continuous image of connected space is connected.

(*b*) A topological space X is locally connected if and only if the components of every open subspace of X are open in X.

7. (*a*) If $\{(X_\alpha, T_\alpha) : \alpha \in \Lambda\}$ is a collection of topological space such that $X = x\ \{X_\alpha : \alpha \in \Lambda\}$, then X is compact relative to the topology if and only if each coordinate space X'_α is compact.

(*b*) Show that the product of two Hausdorff spaces is also a Hausdorff space.

8. (*a*) Prove that the product space $X = x\ \{X_\alpha : \alpha \in \Lambda\}$ is completely regular if and only if each coordinate space X_α is completely regular.

(*b*) Let (X, T) be a product topological space of the topological spaces (X_1, T_1) and (X_2, T_2), then X is compact if and only if X_1 and X_2 both are compact.

9. (*a*) Let (X, T) be a topological space and $Y \subset X$, then $P \in X \Rightarrow P \in \overline{Y}$, if and only if $\exists$ a net in Y converging to P.

(*b*) Let F be a filter on a non empty set X and $A \subset X$, then $\exists$ a filter F^* finer than F such that $A \in F^*$ if and only if $A \cap B \neq \phi \forall B \in F$.

10. State and prove Tietze extension theorem.